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ORIGINAL PAPER

Collective adjustment of pension rights in ALM models

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Abstract Collective adjustment of pension rights is a way to keep defined benefit systems tenable. In asset liability management (ALM) models presented in the literature these decisions are modeled both at the aggregate level of the liabilities as a whole and at a more detailed level. In this paper we compare the approximate aggregate approach to the accurate detailed approach for the average earnings scheme with conditional indexation. We prove that the aggregate approach leads to one-sided errors. Moreover, we show that for semi-realistic data these biases are considerable.

Keywords Asset liability management · Pension funds · Indexation

1 Introduction

In the current economic climate, the risks associated with the pre-millennium defined benefit schemes (DB), where benefit payments are fixed and contributions vary, are regarded to be too high and too unevenly spread among the stakeholders involved. This awareness grew by the severe drop in funding ratios (assets over liabilities) of DB pension funds in the beginning of the new millennium. This was due to two simultaneous developments on the financial markets. On the one hand, the value of the assets of pension funds dropped because of collapsing stock markets. On the other hand, the historically low interest rates resulted in high values for the liabilities. In DB schemes, financial distress can only be averted by raising contributions. Meanwhile, the ageing of the Western populations implies that contribution bases become smaller. In the coming years the population at retirement age over the population at working

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age, is prognosticated to rise sharply. For instance, for the Netherlands this ratio is expected to double the coming 30 years. Besides it's weakening ability of absorbing shocks, the contribution instrument is considered to be unfair, since all pain is felt by the sponsor (company) and active participants (employees) and none by the passive participants (retirees), see e.g. van Ewisk (2005) and Bovenberg and Knaap (2005).

Consequently, DB schemes are abolished or the implementation is altered. In the US and the UK we see a shift to defined contribution (DC) schemes, where contributions are fixed and benefit payments vary. Alternatively, in the Netherlands the prevailing view is to maintain DB based schemes because of the security for the participants. However, the pension rights are no longer fully corrected for inflation each year, which was the case before 2000. The correction for inflation, which is called indexation, is now truly conditional. Moreover, in the Netherlands there is the emergence of the collective defined contribution (CDC) scheme, where rights can be collectively lowered as well. As a consequence, in these schemes the financial position of a pension fund can be actively influenced not only on the asset side via the financing and investment strategies, but on the liabilities side via the adjustment strategy as well. Collectively not granting indexation or lowering the pension rights leads to a lower value of the liabilities. In this paper we focus on the DB scheme with conditional indexation, which is described in detail in Sect. 2. We believe that the results found can be extended to CDC schemes, too.

Strategic asset liability management (ALM) models (both simulation and optimization) are used to support pension fund decisions in the medium and long term. The new situation requires models that include the adjustment policy. Obviously, to be valuable such models must provide a valid abstraction of the adjustment process in practice. In Klein Haneveld et al. (2009) we show that for an accurate modeling of indexation one needs to subdivide the liability related quantities by year of accrual of the underlying rights. Intuitively this can be seen by recalling that indexation is correction for inflation and realizing that the year of accrual determines the inflation the rights have been exposed to. However, in the literature, indexation decisions are generally modeled at the total liabilities level, see e.g. the optimization model of Drijver (2005) and the simulation model of Ponds and van Riel (2007). This aggregation is incorrect, since the proportional contribution of rights accrued in a certain year differs for the various liability related quantities, as we will see in Sect. 4. This information is lost in the aggregation.

The contribution of this paper is a thorough analysis of the errors induced by the aggregate approach. First, a theoretical assessment learns us the nature of the errors. Second, a numerical illustration indicates the sizes of the errors. One should realize that these errors occur at each decision moment considered in the pension fund ALM model. Typically, optimization ALM models constitute around 5 decision moments, whereas simulation ALM models constitute at least 30 decision moments. Therefore, the errors strongly degrade the validity of the models.

The remainder of this paper is organized as follows. Section 2 introduces the buildup of pension rights in average earnings (AE) schemes with conditional indexation. Modeling of conditional indexation in ALM models, which concerns the accurate detailed level approach and the aggregate approximation, is the topic of Sect. 3. In Sect. 4 we theoretically show that the aggregate approximation results in biased errors.

The sizes of these errors are investigated for semi-realistic data in Sect. 5. Finally, Sect. 6 summarizes and touches upon the implications for ALM models.

2 Individual pension rights in AE schemes

In an AE scheme, the rights gained in a certain year are a percentage of the participant's wages for that year. Due to inflation the purchasing power of these rights will diminish with the years. In the traditional Dutch DB setting (before 2000) all rights were corrected fully for inflation each year (though officially it was conditional). This correction for inflation, which typically concerns wage inflation, price inflation, or a combination of both, is called indexation. In the current AE schemes with conditional indexation, indexation is indeed conditional depending on the financial situation of the fund. In general, this indexation process is executed as follows. At the end of year the indexation decision concerning last year's inflation is made. This concerns a factor with which all rights are multiplied. This factor must be at least one, as the rights as accrued are guaranteed, and it must not exceed the inflation factor. Moreover, incomplete indexation, i.e. indexation lower than inflation can be repaired at a later stage. However, such a repair is not retroactive, it only applies to disbursements after the repair. In general, once granted indexation cannot be turned back. Below we give a more detailed description.

Suppose a participant j joins the pension fund in year s_0, then he/she will accrue new rights every year t for $s_0 \leq t \leq t_j(65)$, where $t_j(65)$ denotes the year that the participant turns 65 and retires. Rights accrued in year t in an AE scheme equal the accrual rate, typically 2%, times the pension basis, which is the pensionable wages corrected for the state old-age provision. Denoting the pension basis in year t by P_{jt} the newly accrued rights R^+_{jt} are

$$R^+_{jt} = \begin{cases} aP_{jt} & \text{for } s_0 \leq t \leq t_j(65); \\ 0 & \text{for } t > t(65), \end{cases}$$

where a is the accrual rate. These new rights entitle the participant to be paid the amount R^+_{jt} yearly from $t_j(65)$ till his/her death. The rights as accrued are guaranteed. The total guaranteed rights at time t, which are also called the nominal rights result from summation over all years of accrual:

$$\underline{R}_{jt} = \sum_{s=s_0}^{t} R^+_{js}.$$

Due to inflation the purchasing power of rights accrued in year t reduces every year. The purchasing power is preserved if after accrual the rights are multiplied with the inflation factor every year. Full compensation for inflation leads to what we call fully indexed rights. For the case of compensation for wage inflation these are given by the recursion

$$\overline{R}_{jt} = w_t \overline{R}_{j,t-1} + R^+_{jt}, \tag{1}$$

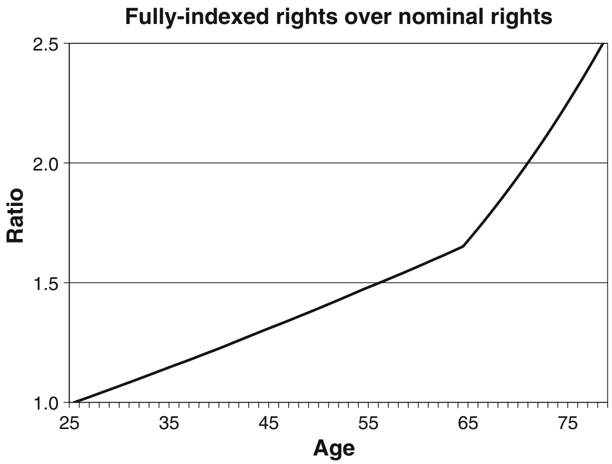

Fig. 1 Fully-indexed rights over nominal rights in an AE scheme for an example working career

where w_t denotes the wage inflation factor for year t. The existing rights are corrected for inflation and the newly accrued rights, which do not need any correction, are simply added. Rewriting (1) totally in terms of the rights as accrued gives

$$\overline{R}_{jt} = \sum_{s=s_0}^{t} W_t^s R_{js}^+,$$

with $W_t^s = \prod_{r=s+1}^{t} w_r$ the accumulated inflation that rights accrued in year s have been exposed to till time t. We let $W_t^t := 1$. Note that we need to consider explicitly all different years of accrual, i.e. only considering the nominal rights is insufficient.

To indicate the magnitude of the differences between the nominal and fully indexed rights, Fig. 1 shows for a stylized working career the ratios of the fully indexed rights over the nominal rights for each time-point of the career. We assume a fixed wage inflation of 3% every year and a career jump every 10 year of 5%. The accrual rate used is 2%. We see that the ratio increases directly with a breaking point at retirement. During the working years the newly accrued rights mitigate the inflation effect.

The actual rights, which determine what the participant is paid during retirement, lie between the nominal rights, which are guaranteed, and the fully indexed rights, which are striven for, and depend on the indexation decisions of the pension fund. At the end of each year, the board of the pension fund decides by how much all old rights will be indexed, with a maximum of the wage inflation of that year. Let ι_t be the *initial indexation* decision for year t, i.e. the nominal rights multiplier, then it must hold that

$$1 \leq \iota_t \leq w_t. \tag{2}$$

Besides initial indexation concerning the last year, it is also possible that past indexation decisions are repaired. The repair applies, however, only to future disbursements. Let $\delta_t^s \geq 0$ be the *repair* at time t of the initial indexation decision taken at time s

with $s < t$. The nonnegativity condition on the repair expresses that once granted indexation cannot be taken back. The *simple indexation decision* for year s at time t ($s < t$), which we denote by i_t^s, equals the initial indexation plus the sum of all repairs:

$$i_t^s = \iota_s + \sum_{r=s+1}^{t} \delta_r^s$$

for which the following conditions hold:

$$1 \le i_t^s \le w_s, \tag{3}$$

$$i_t^s \ge i_{t-1}^s, \tag{4}$$

where (3) is the equivalent of (2), and (4) is implied by the nonnegativity of the repairs.

To calculate the actual value at time t of rights accrued in year s we need to multiply the nominal rights with the indexation decisions at time t for all years in between. So, for each year of accrual s there is a different indexation multiplier, which we call the *accumulated indexation decision* at time t for rights accrued in year s, and is given by

$$I_t^s = \prod_{r=s+1}^{t} i_t^r, \tag{5}$$

with $I_t^t := 1$. The actual rights of participant j at time t are thus given by

$$R_{jt} = \sum_{s=s_0}^{t} I_t^s \, R_{js}^+ \tag{6}$$

Note that we need to consider explicitly all different years of accrual, i.e. only considering the nominal rights is insufficient, as is the case for the fully indexed rights.

3 Conditional indexation and ALM models

ALM models support strategic pension fund decisions, which involves the financial position in the medium and long term. In this setting the value of the total liabilities and the total disbursements resulting from the individual pension rights are of importance. Preferably, the whole ALM model is formulated at this total pension fund level, since including the rights of all individuals explicitly puts a computational burden on the ALM model. In the previous section we saw how individual rights are influenced by indexation decisions. The question now is how to translate this to indexation decisions in an ALM model. In this section we describe the accurate detailed approach and the erroneous aggregate approach introduced before, but not before showing how the value of the liabilities and the disbursements are calculated from the individual pension rights.

All is formulated in a typical discrete time ALM setting. We split the planning horizon in T years, where the resulting decision moments are denoted by an index t. Time $t = 0$ is the current time and $t = T$ is the length of the horizon. Year t ($t = 1, \dots, T$) is the span of time $[t-1, t)$.

3.1 Liabilities and disbursements

In this section it is explained how the total liabilities and disbursements are deducted from given individual rights. Exposure of these relations provides us with important information for the modeling of indexation in ALM models. In our deduction of the liabilities and disbursements we encounter again the three versions nominal, fully indexed, and actual. As before, the first two are predetermined as dictated by the pension contract, whereas the latter depends on yearly decisions by the pension fund board.

Let us first consider the expected benefit payments, which are important building blocks. These are the disbursements the fund expects to make in the future given the present rights, and are calculated on basis of the rights, ages, and survival probabilities of the participants. The expected benefit payment in year $u > t$ is a fraction of the rights present at time t. Letting $e_{jt}(u)$, with $0 \leq e_{jt}(u) \leq 1$, be this fraction for participant j, the nominal expected benefit payment to participant j is

$$\underline{B}_{jt}(u) = e_{jt}(u)\underline{R}_{jt}. \tag{7}$$

The total nominal expected benefit payments follow by summation over all participants:

$$\underline{B}_t(u) = \sum_{j \in J_t} \underline{B}_{jt}(u), \tag{8}$$

where J_t is the set of participants of the fund at time t. The fully indexed and actual expected benefit payments follow by replacing the nominal rights by the fully indexed and actual rights, respectively. Note that aggregation over the participants lets the accrual structure unaffected.

Discounting the expected benefit payments gives the value of the liabilities. So, the value of the nominal liabilities is

$$\underline{L}_t = \sum_{u=t+1}^{U} d_t(u)\underline{B}_t(u), \tag{9}$$

where $U > T$ is the last year of benefit payments as expected at the horizon T and $d_t(u)$ is the discount factor for discounting payments at time u to time t. The values of the fully indexed and actual liabilities follow by discounting the corresponding expected benefit payments. In addition, it is common in ALM studies to distinguish multiple participant groups, e.g. active and passive participants. Therefore we introduce $\underline{L}_{kt}$, $\overline{L}_{kt}$, and L_{kt} as the values of the nominal, fully indexed, and actual liabilities to a participant group k, which depend on the rights owned by the members of the group.

The disbursements in a certain year depend on the rights of the retirees. Let $\underline{R}_t^r$ be the nominal rights of the retirees at time t, then the nominal disbursements in year $t+1$ are

$$\underline{B}_{t+1} = \underline{R}_t^r. \tag{10}$$

The other versions are determined similarly.

In ALM models the liability side of the problem is represented by the value of the liabilities (per participant group) and the disbursements. As until recently all liabilities were fully corrected for inflation every year, the value of the fully indexed liabilities and the fully indexed disbursements were parameters of the models, and the other two versions were not necessary. For the current AE schemes with conditional indexation, the nominal and fully indexed values are parameters, providing lower and upper bounds for the actual values, which are based on the pension fund decisions. In Sect. 2 we discussed how to model this at the individual level. The question is now, how to include indexation decisions at the total pension fund level of ALM models.

3.2 Modeling indexation at the detailed level

For an accurate modeling of indexation it is not necessary to introduce individuals in ALM models, which would lead to tractability problems. It is sufficient to subdivide the value of the liabilities and the disbursements by the years of accrual of the underlying rights. This follows from (6) and the linearities in (7)–(10). Accordingly, we introduce $\underline{L}_t^s$, $\underline{L}_{kt}^s$, and $\underline{B}_t^s$ as the nominal values at time t due to rights accrued in year s. The indexation decisions in the aggregate approach are the accumulated indexation decisions as introduced in (5). The actual values are then

$$L_t^s = I_t^s\, \underline{L}_t^s, \tag{11}$$
$$L_{kt}^s = I_t^s\, \underline{L}_{kt}^s, \tag{12}$$
$$B_{t+1}^s = I_t^s\, \underline{B}_{t+1}^s. \tag{13}$$

The total values are given by the summations over all years of accrual:

$$L_t = \sum_{s=s_I}^{t} L_t^s,$$
$$L_{kt} = \sum_{s=s_I}^{t} L_{kt}^s, \tag{14}$$
$$B_{t+1} = \sum_{s=s_I}^{t} B_{t+1}^s, \tag{15}$$

where s_I is the latest moment that all rights were fully indexed. All rights accrued before s_I can be regarded as accrued in year s_I. In addition to these accounting rules we need accumulated versions of the restrictions on the indexation decisions as given in (3) and (4). These are:

$$I_t^s \leq I_t^{s-1} \leq w_s I_t^s \ , \tag{16}$$

$$\frac{I_t^{s-1}}{I_t^s} \geq \frac{I_{t-1}^{s-1}}{I_{t-1}^s},$$

where we use the fact that $i_t^s = \frac{I_t^{s-1}}{I_t^s}$. This completes the accurate modeling of indexation in ALM models.

The actual value over the nominal value of the liabilities, which we call the *aggregate indexation level*, is a measure for the "total" indexation. Denoting it by I_t it can be written as

$$I_t = \frac{L_t}{\underline{L}_t} = \frac{\sum_{s=s_I}^t L_t^s}{\underline{L}_t} = \frac{\sum_{s=s_I}^t I_t^s \, \underline{L}_t^s}{\underline{L}_t} = \sum_{s=s_I}^t \underline{l}_t^s \, I_t^s, \tag{17}$$

with $\underline{l}_t^s := \underline{L}_t^s / \underline{L}_t$. Observe that $l_t^s \geq 0$ and $\sum_{s=s_I}^t \underline{l}_t^s = 1$, since $\sum_{s=s_I}^t \underline{L}_t^s = \underline{L}_t$. Hence, the aggregate indexation level is a convex combination of the accumulated indexation decisions.

3.3 Modeling indexation at the aggregate level

A drawback of the approach stated above is that it involves quite a number of auxiliary parameters and variables (compared to the fixed liabilities case). An approximate approach with only a minimal amount of extra work is what we call indexation at the aggregate level. Here, the indexation decisions are modeled at the level of the liabilities as a whole, i.e. different years of accrual are not discriminated. The aggregate indexation level I_t, which above is a state variable resulting from the indexation decisions, is now the only decision variable involving indexation. The actual liabilities are thus

$$L_t = I_t \, \underline{L}_t.$$

The approach provides approximations of the value of actual liabilities to specific participant groups and actual disbursements in

$$\hat{L}_{kt} = I_t \, \underline{L}_{kt}, \tag{18}$$

$$\hat{B}_{t+1} = I_t \, \underline{B}_{t+1}. \tag{19}$$

Translation of the constraints (3) and (4) leads in this case to

$$1 \leq I_t \leq \frac{\overline{L}_t}{\underline{L}_t}, \tag{20}$$

$$I_t \geq I_{t-1}. \tag{21}$$

To assess the quality of the aggregate approach, we need to compare the approximate actual values in (18) and (19) and constraint (21) to the accurate counterparts. Note that constraint (20) is valid in the detailed approach as well and cannot lead to errors. The comparison is complicated by the fact that in general an aggregate indexation level I_t can be attained by multiple choices for I_t^s $(s_i \leq s \leq t)$. Nevertheless, in the next section we show that regardless of this choice, the aggregate approach results in one-sided errors.

4 Aggregation errors

In this section we investigate the approximation errors in the aggregate approach. As we will see, these errors result from the difference in proportional contribution of rights accrued in a certain year for the various liability quantities. This leads us to three main results. First, it is shown that the actual disbursements are always underestimated in the aggregate approach. Second, the values of the actual liabilities subdivided into participant groups are wrongly calculated. Third, the lower bound on next year's indexation implied by the indexation process is too tight in the aggregate approach. In addition to these biases, the aggregate approach leaves a fixed distribution of the indexation over the years of accrual, which is too confining.

4.1 Modus operandi

To assess the quality of the aggregate approach we adopt the following method. For given detailed indexation decisions I_t^s, where s ranges from s_I to t, we compare the correct values of the relevant variables/constraints to the approximations obtained when using the corresponding aggregate indexation level I_t. Application of the following proposition proves to be useful.

Proposition 1 *Let $p, q, x \in \mathbb{R}^n$ be given such that*

1. *x_j is monotonous non-increasing in j and $x_1 > x_n$;*
2. *$p_j, q_j \geq 0$ for all j and $\sum_{j=1}^n p_j = \sum_{j=1}^n q_j = 1$;*
3. *there is a k with $1 \leq k < n$ such that $p_j > q_j$ for $1 \leq j \leq k$ and $p_j < q_j$ for $k + 1 \leq j \leq n$.*

Then it holds that

$$\sum_{j=1}^n p_j x_j > \sum_{j=1}^n q_j x_j.$$

Proof Introducing $a_j := p_j - q_j$, proving the above amounts to proving $\sum_{j=1}^{n} a_j x_j > 0$. We have that

$$\begin{aligned}\sum_{j=1}^{n} a_j x_j &= a_1 x_1 + \sum_{j=2}^{k} a_j x_j + \sum_{j=k+1}^{n-1} a_j x_j + a_n x_n \\ &\geq a_1 x_1 + \sum_{j=2}^{k} a_j x_k + \sum_{j=k+1}^{n-1} a_j x_k + a_n x_n \qquad (22) \\ &= a_1 x_1 + \sum_{j=2}^{n-1} a_j x_k + a_n x_n \\ &= a_1 x_1 + (-a_1 - a_n) x_k + a_n x_n \qquad (23) \\ &= a_1 (x_1 - x_k) + a_n (x_n - x_k) \\ &> 0, \qquad (24)\end{aligned}$$

where (22) follows from (a) and $a_j > 0$ for $j \leq k$ and $a_j < 0$ for $j > k$, (23) from $\sum_{j=1}^{n} a_j = 0$, and (24) from $x_1 > x_k$ or $x_k > x_n$ due to (a) and $a_1 > 0$ and $a_n < 0$. □

We can interpret Proposition 1 as follows. The x_i can be considered as realizations of a discrete random variable X, and the p_j and q_j as elements from probability measures $\mathbb{P}$ and $\mathbb{Q}$. The probability measure $\mathbb{P}$ gives the higher values of x more mass than $\mathbb{Q}$. As a result,

$$\mathbb{E}_{\mathbb{P}}[X] > \mathbb{E}_{\mathbb{Q}}[X].$$

The pertinence of Proposition 1 in our indexation context follows from noting that the decisions I_t^s at time t, where s ranges from s_I to t, satisfy condition (1) for aggregate indexation levels $I_t > 1$, which we call non-trivial indexation. The fact that I_t^s is non-increasing in s, follows from the first inequality of (16). Recall that $I_t^t = 1$, as the just accrued rights cannot be indexed. Since I_t is a convex combination of the non-increasing I_t^s, it follows that for $I_t > 1$ it holds that $I_t^{s_I} > 1$ and thus $I_t^{s_I} > I_t^t$. Furthermore, in the examination of the aggregate approach we will face weights that together with $\underline{l}_t^s$ satisfy conditions (2) and (3), as the actual values over the nominal values of other quantities than the liabilities are convex combinations of the detailed decisions as well. In this way, we can prove that the aggregate approach leads to biases.

4.2 Biased actual disbursements

The indexation at time t determines the value of the actual disbursements in year $t+1$. The difference between the actual disbursements in the aggregate and the detailed model is

$$\hat{B}_{t+1} - B_{t+1}, \qquad (25)$$

which we call the *disbursement error*. Application of the aggregate model definition (19) gives

$$\hat{B}_{t+1} = I_t \, \underline{B}_{t+1} = \underline{B}_{t+1} \sum_{s=s_I}^{t} \underline{l}_t^s \, I_t^s,$$

where the last equality follows from $I_t = \sum_{s=s_I}^{t} \underline{l}_t^s I_t^s$, see (17). In the detailed model equations (13) and (15) give

$$B_{t+1} = \sum_{s=s_I}^{t} I_t^s \, \underline{B}_{t+1}^s = \underline{B}_{t+1} \sum_{s=s_I}^{t} \frac{I_t^s \, \underline{B}_{t+1}^s}{\underline{B}_{t+1}} = \underline{B}_{t+1} \sum_{s=s_I}^{t} \underline{b}_{t+1}^s I_t^s,$$

where $\underline{b}_{t+1}^s := \underline{B}_{t+1}^s / \underline{B}_{t+1}$ with $\underline{b}_{t+1}^s \geq 0$ for all s and $\sum_{s=s_I}^{t} \underline{b}_{t+1}^s = 1$. Consequently, the disbursement error equals

$$\underline{B}_{t+1}^s \sum_{s=s_I}^{t} (\underline{l}_t^s - \underline{b}_{t+1}^s) I_t^s.$$

The ratios $\underline{b}_{t+1}^s$ and $\underline{l}_t^s$ are, respectively, the parts of the nominal disbursements and nominal liabilities accounted for by rights of year s. Whereas $\underline{b}_{t+1}^s$ only concerns the payments in year $t+1$, $\underline{l}_t^s$ concerns all remaining (discounted) expected benefit payments. As older rights have less remaining benefit payments than newer rights, for older rights, i.e. s close to s_I, it holds that $\underline{b}_{t+1}^s > \underline{l}_t^s$, while for newer rights, i.e. s close to t, it holds that $\underline{b}_{t+1}^s < \underline{l}_t^s$. Consequently, it is plausible to assume the following.

Property 1 There is an index $\hat{s}$ ($s_I \leq \hat{s} < t$) such that $\underline{b}_{t+1}^s > \underline{l}_t^s$ for $s \leq \hat{s}$, and $\underline{b}_{t+1}^s < \underline{l}_t^s$ for $s > \hat{s}$.

Application of Proposition 1 gives that for non-trivial indexation

$$\sum_{s=s_I}^{t} \underline{b}_{t+1}^s \, I_t^s > \sum_{s=s_I}^{t} \underline{l}_t^s \, I_t^s,$$

which gives that

$$\underline{B}_{t+1}^s \sum_{s=s_I}^{t} (\underline{l}_t^s - \underline{b}_{t+1}^s) I_t^s < 0.$$

Hence, the disbursement error is negative for all non-trivial indexation decisions. Put differently, indexation at the aggregate level results in underestimation of the actual disbursements.

4.3 Incorrect values of actual liabilities to participant groups

The indexation at time t determines the value of the actual liabilities to the participant groups at time t. The difference in these values in the aggregate and the detailed model is

$$\hat{L}_{kt} - L_{kt},$$

which we call the *participant group liabilities error*. The definition at the aggregate level, as specified in (18) can be rewritten as

$$\hat{L}_{kt} = I_t\, \underline{L}_{kt} = \underline{L}_{kt} \sum_{s=s_I}^{t} \underline{l}_t^s\, I_t^s,$$

where the last equality follows from $I_t = \sum_{s=s_I}^{t} \underline{l}_t^s I_t^s$, see (17). Application of the definition at the detailed level, as specified in (12) and (14), results in

$$L_{kt} = \sum_{s=s_I}^{t} I_t^s\, \underline{L}_{kt}^s = \underline{L}_{kt} \sum_{s=s_I}^{t} \frac{I_t^s\, \underline{L}_{kt}^s}{\underline{L}_{kt}} = \underline{L}_{kt} \sum_{s=s_I}^{t} \underline{l}_{kt}^s\, I_t^s,$$

where $\underline{l}_{kt}^s = \underline{L}_{kt}^s / \underline{L}_{kt}$ with $\underline{l}_{kt}^s \geq 0$ for all s and $\sum_{s=s_I}^{t} \underline{l}_{kt}^s = 1$. Consequently, the participant group liabilities error equals

$$\underline{L}_{kt} \sum_{s=s_I}^{t} (\underline{l}_t^s - \underline{l}_{kt}^s) I_t^s.$$

In Klein Haneveld et al. (2009), we consider a division into active ($k = 1$) and passive participants ($k = 2$). Active participants own the largest proportions of the newer rights, whereas passive participants own the largest proportions of the older rights. For this reason we assume the following for the active participants.

Property 2 There is an index s_1 ($s_I \leq s_1 \leq t$) such that $\underline{l}_{1t}^s < \underline{l}_t^s$ for $s \leq s_1$, and $\underline{l}_{1t}^s > \underline{l}_t^s$ for $s > s_1$.

Application of Proposition 1 gives for non-trivial indexation

$$\sum_{s=s_I}^{t} \underline{l}_{1t}^s I_t^s < \sum_{s=s_I}^{t} \underline{l}_t^s I_t^s,$$

which implies for the participant group liabilities error for $k = 1$

$$\underline{L}_{1t} \sum_{s=s_I}^{t} (\underline{l}_t^s - \underline{l}_{1t}^s) I_t^s > 0. \tag{26}$$

Hence, the actual liabilities error for the active participants, which we call the *active liabilities error*, is positive for all non-trivial indexation decisions. Put differently, the value of the actual liabilities to active participants is thus overestimated in the aggregate approach.

For the passive participants we generally have the following.

Property 3 There is an index s_2 ($s_I \leq s_2 \leq t$) such that $\underline{l}^s_{2t} > \underline{l}^s_t$ for $s \leq s_2$, and $\underline{l}^s_{2t} < \underline{l}^s_t$ for $s > s_2$.

Using Proposition 1 we get for non-trivial indexation

$$\sum_{s=s_I}^{t} \underline{l}^s_{2t} I^s_t > \sum_{s=s_I}^{t} \underline{l}^s_t I^s_t,$$

which implies for the participant group liabilities error for $k = 2$

$$\underline{L}_{2t} \sum_{s=s_I}^{t} (\underline{l}^s_t - \underline{l}^s_{2t}) I^s_t < 0. \tag{27}$$

Hence, the actual liabilities error for the passive participants, which we call the *passive liabilities error*, is negative for all non-trivial indexation decisions. Put differently, the value of the actual liabilities to passive participants is thus underestimated in the aggregate approach.

4.4 Biased lower bound on next year's indexation

The indexation at the next decision moment is intertwined with the indexation at the current decision moment, since once granted indexation cannot be turned back. More specifically, the indexation at time t determines a lower bound on the indexation at time $t+1$. The lower bound at time $t+1$ in the aggregate approach, which we denote by $\hat{I}^{lb}_{t+1}$, is given in (21), i.e.

$$\hat{I}^{lb}_{t+1} = I_t = \sum_{s=s_I}^{t} \underline{l}^s_t I^s_t.$$

At the detailed level, for each year of accrual it must hold that

$$I^s_{t+1} \geq I^s_t,$$

which follows from $i^s_{t+1} \geq i^s_t$, $i^{t+1}_{t+1} \geq 1$, and $I^s_t = \prod_{r=s+1}^{t} i^r_t$. Hence, the lower bound on the aggregate indexation level in the detailed approach can be found by letting $I^s_{t+1} = I^s_t$ for $s_I \leq s \leq t$ and $I^{t+1}_{t+1} = 1$ (by definition, see (5)). The resulting lower bound, which we denote by I^{lb}_{t+1}, is given by

$$I_{t+1}^{lb} = \frac{\sum_{s=s_I}^{t} I_t^s \underline{L}_{t+1}^s + \underline{L}_{t+1}^{t+1}}{\underline{L}_{t+1}} = \sum_{s=s_I}^{t} \underline{l}_{t+1}^s I_t^s + \underline{l}_{t+1}^{t+1}$$

The difference in lower bound for the aggregate and detailed approach is

$$\hat{I}_{t+1}^{lb} - I_{t+1}^{lb}, \tag{28}$$

which we call the *lower bound error*. This error can be rewritten to

$$\sum_{s=s_I}^{t} (\underline{l}_t^s - \underline{l}_{t+1}^s) I_t^s - \underline{l}_{t+1}^{t+1}.$$

Conveniently introducing $I_t^{t+1} = 1$ and $\underline{l}_t^{t+1} = 0$, the difference in bounds becomes

$$\sum_{s=s_I}^{t+1} (\underline{l}_t^s - \underline{l}_{t+1}^s) I_t^s. \tag{29}$$

About the relationship between $\underline{l}_t^s$ and $\underline{l}_{t+1}^s$ we know that $\underline{l}_{t+1}^{t+1} > \underline{l}_t^{t+1} = 0$. Moreover, for the older rights $\underline{l}_t^s$ will be larger than $\underline{l}_{t+1}^s$, because the benefit payments in year $t+1$ that were part of $\underline{l}_t^s$ are no part of $\underline{l}_{t+1}^s$. This leads to the following assumption.

Property 4 There is an index $\hat{s}$ $(s_I \leq \hat{s} < t)$ such that $\underline{l}_t^s > \underline{l}_{t+1}^s$ for $s \leq \hat{s}$ and $\underline{l}_t^s < \underline{l}_{t+1}^s$ for $s > \hat{s}$.

Application of Proposition 1 gives that for non-trivial indexation expression (29) is greater than zero. Hence, the lower bound error is positive, for all non-trivial indexation decisions. Put differently, the lower bound on the aggregate indexation level at time $t+1$ is thus overestimated when indexing at the aggregate level.

4.5 Fixed indexation distribution

So far, we only discussed ‘accounting’ errors resulting from the use of a single aggregate indexation decision. We paid little attention to the fact that a certain aggregate indexation level can be attained by multiple choices for the detailed indexation decisions. However, these different choices do lead to different outcomes. Concerning the participants, in Klein Haneveld et al. (2009) we show that active participants prefer the newest rights to be indexed, i.e. the most recent wage inflation to be corrected, whereas passive participants prefer the oldest rights to be indexed. Moreover, different choices result in different disbursement patterns and different lower bounds on the indexation at the next decision moment. Indexation at the aggregate level does not allow for these choices, as there is only one indexation decision for each decision moment, which further invalidates the aggregate approach.

5 Numerical illustration

In the previous section, we proved that the aggregate approach to indexation leads to biases. To which degree this happens is investigated here. For semi-realistic data, as obtained from ORTEC (http://www.ortec.com), we calculate the aggregate errors as indicated in Sects. 4.2 to 4.4. The choice aspect of Sect. 4.5 is reflected in the use of two choices for the detailed decisions for each aggregate level considered, namely one choice reflecting the interests of the active participants and one reflecting the interests of the passive participants. The magnitude of the errors found is considerable and increases with the number of accrual years that need to be taken into account.

5.1 Data

We consider the fictitious Hollandia pension fund, which is constructed by ORTEC to represent a typical Dutch pension fund. We distinguish two participant groups, namely the active ($k = 1$) and the passive participants ($k = 2$). The latest moment that all liabilities were fully indexed is fixed to ultimo 2000, i.e. $s_I = 2000$. Consequently, all rights accrued before 2000 can and are regarded to be accrued in 2000. To investigate the effect of the number of years of accrual that need to be taken into account, we evaluate two decision moments, namely ultimo 2006 ($t = 2006$) and ultimo 2014 ($t = 2014$). Thus, for $t = 2006$ there are 7 different years of accrual, while for $t = 2014$ there are 15 different years of accrual. The required data are listed in Appendix A, where Table 3 concerns $t = 2006$ and Table 4 concerns $t = 2014$. The pension liability data are provided by ORTEC Finance bv and generated with the Pension Assets & Liabilities scenario Model (PALM) (Boender 1997; Boender et al. 1998, 2007). The wage increase factors for years 2001 to 2006 stem from Statistics Netherlands (http://www.cbs.nl). From 2007 onwards, the factors equal the average Dutch yearly wage increase factor.

A first step in our numerical analysis is to check whether Properties 1 to 4 hold for the data. The involved convex weights $\underline{l}_t^s$, $\underline{b}_{t+1}^s$, $\underline{l}_{1t}^s$, $\underline{l}_{2t}^s$, and $\underline{l}_{t+1}^s$ follow from Tables 3 and 4 by straightforward computation. To investigate the assumed relations, where $\underline{l}_t^s$ is compared to the other weights, we list in Table 1 the differences of the other weights with $\underline{l}_t^s$ for $t = 2006$. We see directly that all differences have only one switch from positive to negative or vice versa. Moreover, the patterns match with the assumed relations. For instance, the index $\hat{s}$ mentioned in Property 1 equals 2000. A similar analysis of the data for $t = 2014$ confirms the assumed relations as well.

5.2 Results

In this section, we present the aggregate errors as defined in equations (25) and (26)–(28) for the decision moments $t = 2006$ and $t = 2014$. To take into account the effect of the height of indexation, the errors are calculated for different levels of *relative indexation*, which is given by

$$\frac{L_t - \underline{L}_t}{\overline{L}_t - \underline{L}_t}.$$

Table 1 Difference with $\underline{l}^s_{2006}$

s	$\underline{b}^s_{2007}$	$\underline{l}^s_{1,2006}$	$\underline{l}^s_{2,2006}$	$\underline{l}^s_{2007}$
2000	0.160	−0.126	0.131	−0.035
2001	−0.021	0.008	−0.008	−0.001
2002	−0.023	0.013	−0.013	−0.001
2003	−0.025	0.017	−0.018	−0.001
2004	−0.027	0.023	−0.024	−0.001
2005	−0.031	0.029	−0.030	−0.001
2006	−0.034	0.037	−0.038	−0.001
2007	–	–	–	0.040

Note that the relative and aggregate indexation level are equivalent, as $L_t = I_t \underline{L}_t$. A relative indexation of 0% belongs to the nominal situation, whereas 100% is the fully indexed case. We calculated the aggregate errors for relative indexation levels of 0, 10, 20, ..., 100%.

The detailed approach is represented by two choices for each level of relative/aggregate indexation (recall that, in general, an aggregate indexation level can be attained by multiple detailed decisions). These two choices are related to the two participant groups. As mentioned in Sect. 4.5 active participants prefer correction of the newest inflation, whereas passive participants prefer correction of the oldest inflation. One choice for the detailed decisions is what we call *active indexation decisions*, which are found by first correcting the most recent inflation, next the one but most recent inflation, etc. This is continued until the relative indexation level is reached. The other choice for the detailed decisions we consider are the *passive indexation decisions*, which are found the opposite way, thus starting with the oldest not yet corrected wage inflation.

5.2.1 Decision moment $t = 2006$

Let us first have a look at the actual disbursements. In the aggregate approach, the actual disbursements in a year are the nominal disbursements multiplied with the aggregate indexation level. From Sect. 4.2 we know that this leads to actual disbursements that are too low, irrespective of the particular detailed decisions accounting for the aggregate level. In Table 2 the actual disbursements in both approaches for different levels of relative indexation are given. The second column concerns the aggregate approach, whereas the third and fourth column concern the active and passive decisions in the detailed approach. The fourth column expresses the disbursement error as a percentage of the actual disbursements for active detailed decisions. The sixth column expresses the same for the passive detailed decisions. As expected from Sect. 4, we observe that for non-trivial indexation levels the aggregate approach gives too low actual disbursements, irrespective which of the two detailed choices is considered. Note the logical outcomes that no indexation results in the same actual disbursements for both approaches, and that in case of full indexation the actual disbursements for active and passive detailed decisions are equal.

The disbursement errors together with the other three errors are displayed in Fig. 2. The horizontal axis represents the relative indexation, whereas the vertical axis

Table 2 Actual disbursements for $t = 2006$ (in thousands of Euros)

Indexation (%)	Aggregate	Active detailed		Passive detailed	
0	77,845	77,845	0.00%	77,845	0.00%
10	78,959	79,011	−0.07%	79,201	−0.31%
20	80,073	80,224	−0.19%	80,558	−0.60%
30	81,186	81,455	−0.33%	81,891	−0.86%
40	82,300	82,726	−0.51%	83,204	−1.09%
50	83,414	83,997	−0.70%	84,516	−1.30%
60	84,527	85,310	−0.92%	85,788	−1.47%
70	85,641	86,622	−1.13%	87,059	−1.63%
80	86,755	87,956	−1.37%	88,289	−1.74%
90	87,868	89,312	−1.62%	89,502	−1.83%
100	88,982	90,668	−1.86%	90,668	−1.86%

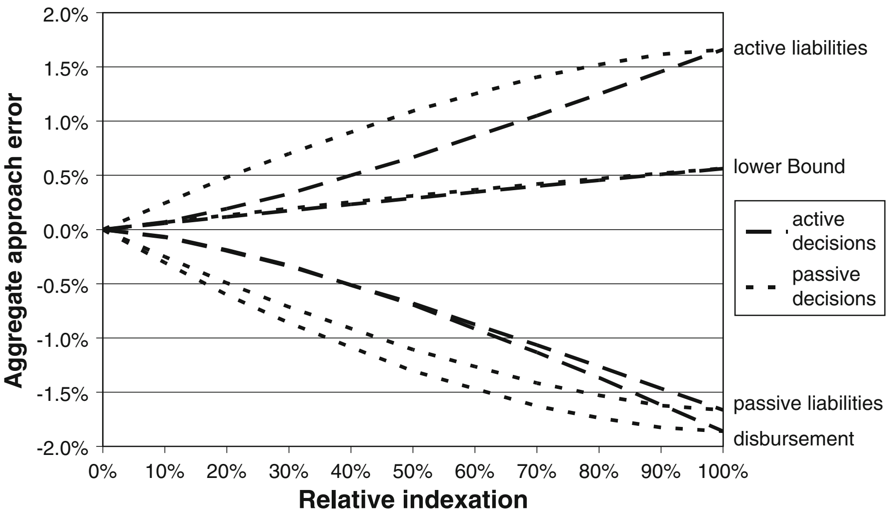

Fig. 2 Aggregate approach errors for $t = 2006$ and $s_I = 2000$

displays the aggregate approach errors. The four groups indicated are the different types of errors considered. For each group two situations are distinguished, namely active detailed decisions and passive detailed decisions. Hence, the upper most line is the aggregate approach error for the value of the active liabilities in the case of passive detailed decisions. The line just beneath it is that error for the case of active detailed decisions. The lines result from connecting the observations for the eleven levels of relative indexation (as specified in the first column of Table 2).

The figure tells us that all errors have the sign as expected from Sect. 4: the active liabilities and lower bound error is positive, and the passive liabilities and disbursement error is negative. Moreover, the sizes of the errors are proportional to the level of relative indexation. For the disbursements, active liabilities, and passive liabilities errors we observe two striking patterns. First, the errors for passive detailed decisions are larger than those for active detailed decisions. This is a result of the fact that for passive detailed decisions, the oldest rights have priority over the newer rights. As the difference in weights is by far the biggest for the oldest rights for the three quantities considered (see the second row of Table 1), the errors are larger in the passive detailed case. A second observation is that for active detailed decisions the errors deteriorate the quickest at high levels of relative indexation, whereas for passive detailed decisions the errors deteriorate the quickest at low levels of relative indexation. This can be explained as follows. Consider some additional indexation in the active detailed case. For low levels of relative indexation, emphasizing correction of the most recent inflation results in additional indexation of almost all rights, see equations (5) and (11), which is close to what the aggregate approach does (additionally index all rights with the same factor). However, the higher the indexation level, the older the wage inflation that is corrected, the less rights that are corrected by the additional indexation, the larger the gap with the aggregate approach. For the passive detailed decisions the opposite holds.

5.2.2 Decision moment $t = 2014$

To investigate the effect of more years of accrual considered, we generated the aggregate errors for the decision moment $t = 2014$, while keeping the same latest moment of full indexation ($s_I = 2000$). The number of years of accrual is about doubled compared to $t = 2006$. The results are represented in Fig. 3, which is constructed in the same way as Fig. 2. The errors are more extreme than in case of $t = 2006$, while the patterns are the same. Looking at full indexation (relative indexation of 100%) we see that the lower bound error about doubles, the disbursements and passive liabilities error about triples, and the active liabilities error about quadruples.

6 Concluding remarks

Collective adjustment of pension rights is a way to keep the DB system tenable. An important pension scheme with collective adjustment is the AE scheme with conditional indexation. For an accurate modeling of conditional indexation in ALM models, the liabilities need to be subdivided according to the years in which the underlying rights were accrued. However, in the literature indexation is generally modeled at the total liabilities level, which is to be seen as an approximate approach. We proved for the AE scheme with conditional indexation that application of this approximate approach leads to biases in the disbursements, the value of the liabilities to participant groups, and in the lower bound on the adjustment for the next decision moment. These biases occur each decision moment considered. A numerical analysis of these biases for semi-realistic data showed that the biases are considerable and increase with the number of years of accrual that need to be considered. Moreover, different choices for the detailed decisions that result in the same aggregate level, lead to different biases.

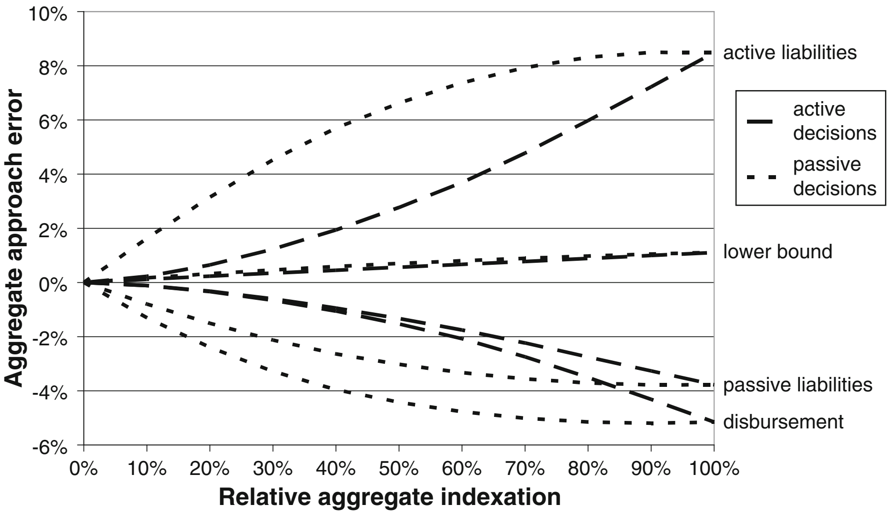

Fig. 3 Aggregate approach errors for $t = 2014$ and $s_I = 2000$

To which extent the validity of an ALM study based on the aggregate approach is adversely affected by these biases mainly depends on the characteristics of the ALM model and the pension fund considered. Concerning the first, long horizons, which are typical for ALM models result in more years of accrual to be considered, and thus in larger biases. Especially for long term simulation models this is a point of attention. Concerning the latter, e.g., the less homogeneous the fund considered, the larger the bias in the valuation of the liabilities to participant groups. In any case, when constructing an ALM model that includes adjustment decisions, one should be aware of the drawbacks of the aggregate approach.

Appendix A: A Pension fund data

Table 3 Values of parameters for the decision moment $t = 2006$, all in thousands of Euros except for the wage inflation factors

s	$\underline{L}^s_{2006}$	$\underline{L}^s_{1,2006}$	$\underline{L}^s_{2,2006}$	$\underline{B}^s_{2007}$	$\underline{L}^s_{2007}$	w_s
2000	1,572,053	662,707	909,346	69,496	1,566,341	–
2001	95,699	57,422	38,277	1,844	97,798	1.043
2002	96,134	63,110	33,025	1,732	98,332	1.043
2003	98,625	68,744	29,882	1,619	101,041	1.033
2004	97,135	74,210	22,925	1,458	99,545	1.025
2005	94,707	79,832	14,875	1,043	97,504	1.012
2006	90,088	85,730	4,358	653	93,074	1.010
2007	–	–	–	–	88,826	1.030
Total	2,144,441	1,091,754	1,052,687	77,845	2,242,463	–

Table 4 Values of parameters for the decision moment $t = 2014$, all in thousands of Euros except for the wage inflation factors

s	$\underline{L}^s_{2014}$	$\underline{L}^s_{1,2014}$	$\underline{L}^s_{2,2014}$	$\underline{B}^s_{2015}$	$\underline{L}^s_{2015}$	w_s
2000	1,369,438	191,199	1,178,240	89,663	1,330,094	–
2001	102,228	23,353	78,875	4,421	101,672	1.043
2002	103,469	26,522	76,947	4,230	103,114	1.043
2003	107,555	29,781	77,775	4,224	107,414	1.033
2004	107,106	32,807	74,300	4,064	107,160	1.025
2005	107,647	36,169	71,478	3,799	108,028	1.012
2006	104,621	39,629	64,992	3,402	105,175	1.010
2007	103,697	44,406	59,291	2,915	104,918	1.030
2008	104,478	49,495	54,983	2,617	106,066	1.030
2009	103,989	55,144	48,845	2,274	105,882	1.030
2010	103,190	61,513	41,677	1,792	105,537	1.030
2011	101,696	68,538	33,158	1,390	104,352	1.030
2012	103,557	76,308	27,249	1,138	106,523	1.030
2013	101,476	84,831	16,644	795	104,705	1.030
2014	100,712	94,275	6,437	448	104,300	1.030
2015	–	–	–	–	101,424	1.030
Total	2,824,858	913,968	1,910,890	127,171	2,906,364	–

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